Exercise 4

Solve the differential equation.

$$y'' + 8y' + 16y = 0$$

Solution

This is a linear homogeneous ODE with constant coefficients, so it has solutions of the form $y = e^{rx}$.

$$y = e^{rx} \quad \rightarrow \quad y' = re^{rx} \quad \rightarrow \quad y'' = r^2 e^{rx}$$

Substitute these formulas into the ODE.

$$r^2 e^{rx} + 8(re^{rx}) + 16(e^{rx}) = 0$$

Divide both sides by e^{rx}.

$$r^2 + 8r + 16 = 0$$

Solve for r.

$$(r+4)^2 = 0$$

$$r = \{-4\}$$

Two solutions to the ODE are e^{-4x} and xe^{-4x}. According to the principle of superposition, the general solution to the ODE is a linear combination of these two.

$$y(x) = C_1 e^{-4x} + C_2 x e^{-4x}$$

C_1 and C_2 are arbitrary constants.